

## CS401 Final Exam — Aug 2006

**Instructions:** Answer all five (5) questions. The three you do best on will be worth 25 points each. The one you do fourth-best on will be worth 15 points. The one you do fifth-best on will be worth 10 points. Please show any relevant calculations. (And be sure to include your name!)

- (I) Every day your aunt takes four pills: two red ones, one green one, and one blue one. If eaten by a dog, each pill produces certain symptoms one hour later, with certain probabilities:

Pill	$P(\text{blue tongue})$	$P(\text{tremors})$	$P(\text{trots})$
R	0.2	0.7	0.1
G	0.8	0.3	0.25
B	0.5	0.4	0.05

One morning your dog snatches a pill from your aunt's hand and swallows it as she takes the first of her four pills, chosen at random from a cup containing the day's four pills. The remaining three pills in the cup fall down the drain, and cannot be recovered. One hour later, your dog has a blue tongue, does not have tremors, and does not have the trots. Given all the above information, calculate the posterior probability distribution of which pill the dog has swallowed:

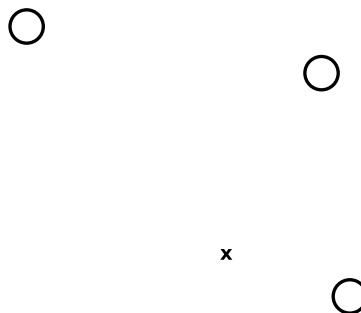
$$P(\text{pill} = \text{R} \mid \text{data}) = ?$$

$$P(\text{pill} = \text{G} \mid \text{data}) = ?$$

$$P(\text{pill} = \text{B} \mid \text{data}) = ?$$

(Carefully show your calculations. Also describe at least one "reality check" on your calculations, and show that it is met.)

- (II) In the diagram below, the open circles correspond to cluster centres for a  $k$ -means algorithm. Show which cluster centre(s) would move, and in which direction(s), if presented with the input point marked  $x$ .



- (III) Define the *maximum margin separating hyperplane*, and give an example showing the separation surface and the margin in a 2D input space with three input points, arranged in an equilateral triangle, with two of the points in one class and the third in the other.

- (IV)** Consider the input space of assignments of values to the ten boolean variables  $(v_0, \dots, v_9)$  and the concept class of disjunctions of three variables. For instance, one element of this concept class would be:  $v_2$  or  $v_3$  or  $v_7$

What is the VC dimension of this concept class? (Reminder: this is the maximum number of input points that can be “shattered” by this concept class.)

- (V)** Consider a standard two-layer multilayer perceptron, with a two dimensional input space and two hidden units. The units each have a “hard” -1/+1 threshold. Each of the three units (the two hidden units and the output unit) has three free parameters: the weights on its two inputs, and its bias. Draw such a network, and assign values to the free parameters so that the network generates the given output for the inputs in the table below.

input <sub>1</sub>	input <sub>2</sub>	output
-1	-1	-1
+1	-1	+1
0	0	-1
-1	+1	+1
+1	+1	-1